

OPTIMAL RETURN IN A MODEL OF BANK SMALL-BUSINESS FINANCING

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Abstract

This paper develops a model of small business finance in which funding constraints in the banking sector affect the access and cost of external finance of firms. Banks attract funds from a large number of small investors who are prone to coordination problems and lend them to small, entrepreneurial firms. We employ a global games methodology in which random technological costs of firm's investment projects pin-down a unique equilibrium to the investors' coordination problem. We show how banks can mitigate these coordination problems and increase the firm's access to capital at a higher interest rate than one might expect under a perfect competitive environment. Paying a higher interest rate is in the interest of the firm, because it relaxes the funding constraint.

Keywords: Small business financing, bank finance, global games, switching equilibrium, optimal return.

JEL Classification index: D82; G21; G32.

1 Introduction

In modern market economies small businesses are an important vector of growth, job-creation and innovation. By contrast with large firms, small firms are more “informational opaque” insofar as they do not disclose certified risk rates or credit scores and do not publish systematically audited financial statements or

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data on collateral (Berger and Udell, 2006). As a consequence, access to external capital is an essential growth constraint of small, entrepreneurial firms (Beck and Demirguc-Kunt, 2006). According to a large consensus in economic literature, bank relationship-lending has emerged as an efficient way of overcoming these informational frictions characterizing small firms (Petersen and Rajan, 1994). Unsurprisingly, small and medium enterprises (SMEs) are largely dependent on banks for their external finance. For example, taking stock from survey data on Western European countries, Berger and Schaeck (2011) find that 60% of the SMEs surveyed rely on bank-financing, with most funds coming from small, regional financial institutions.¹

Banks play a key intermediation role between small firms on the one hand, and investors on the other hand. A large literature has focused on the role of banks in facilitating the access to credit of small, informational opaque borrowers by engaging in “relationship lending” (see Boot, 2000, for an overview). Through this strong firm-creditor relationship, the bank can acquire specific information about the firm, such as the entrepreneur’s talent, future prospects and business environment, which generally leads to an increased availability of credit to small firms (Petersen and Rajan, 1994; Berger and Udell, 1995; Cole, 1998; Bartoli, Ferri, Murro and Rotondi, 2013). The effect of this type of lending on the cost of capital is, however, less clear. From a theoretical standpoint, a closer bank-firm relationship should reduce the bank’s costs of acquiring information, and thus reduce the loan risk premium. If competition in the banking sector is strong, these cost savings should be passed on to borrowers as lower interest rates (Petersen and Rajan, 1994; Boot and Thakor, 1994). However, if the “soft” information specific to bank lending relationships cannot be accessed easily by external investors, then the bank acquires some form of “informational market power” that might justify a higher interest rate (as in Greenbaum, Kanatas and Venezia, 1989; Rajan, 1992). So far the empirical literature has not reached a definitive conclusion on this important issue.²

A related literature has focused on the bank-to-investor relationship. In general loans to opaque and risky small businesses cannot be used by banks as a collateral. Thus banks must raise funds from those clients who are interested in

¹They also find that half of the firms that rely of bank funding obtain it from a single bank.

²Looking at US data, Berger and Udell (1995) find that strong bank-borrower relationships are empirically associated with lower loan interest rates, while Petersen and Rajan (1994) find no statistically significant link. Taking evidence from European data, Degryse and Van Cayseele (2000) show that loan rates actually increase in the duration of the relationship, while Harhoff and Körting (1998) find no effect.

such projects. Actually, substantial empirical evidence has shown that smaller-sized, local banks are more likely to lend to small, less transparent firms (see Berger and Udell, 2002; Scott, 2004; Berger and Black, 2011). At the same time, these small banks are also more likely to be constrained in their own access to external finance (for empirical evidence see Paravisini, 2008; Banerjee and Duflo, 2008; Khwaja and Mian, 2008; Iyer, Peydr, da Rocha-Lopes and Schoar, 2014).

A thorough analysis of small-business bank finance must aggregate the two sides of the market, with banks playing a pivotal role between firms and investors. This paper aims at building a model of small business finance that takes into account both a technological risk specific to small, entrepreneurial firms and a particular financial risk stemming from a would-be inability of investors to coordinate their investment decisions. It emphasizes the importance of funding constraints for banks themselves and how they can affect the availability and cost of capital for small entrepreneurial firms. We show that equilibrium interest rates on loans are higher than one might expect under a perfectly competitive banking sector since higher interest rates mitigate the coordination problem arising among investors.³

We model a competitive financial sector in which bank profits tend to zero, i.e. interest rates on loans equal those on deposits. Yet, this does not mean that the bank is a passive player. It still decides on this interest rate, given a specific goal. In this paper, we assume that the bank aims to maximize the chances that the entrepreneurial firm succeeds in implementing its project. This "pragmatic" goal can be justified in several ways. First, it is consistent with the bank aiming to build a strong reputation, which is the most important asset in a competitive environment and also a criterion for the bank's survival in the long run. Second, banks often use a successful first loan arrangement as a way of engaging in a profitable long-term relationship; once "caught" as a client, the firm is offered a broader range of services, such as checking and savings accounts or trade finance (Degryse and Van Cayseele, 2000; Berger and Udell, 2002).⁴

The real sector in our model is represented by a small entrepreneurial firm who owns an innovative, risky technology and seeks to raise capital to implement

³We chose on purpose the framework of a competitive banking sector, to rule out any "market power" effect that would justify a high interest rate by itself.

⁴A model where the bank maximizes its profit, and sets different interest rates on loans and deposits can be developed along the same lines. The version presented in this paper is to be preferred on the Occam's Razor principle, i.e. to prove the key result with least analytical complexity.

it. The firm’s output is increasing in the amount of capital according to a linear technology. Technological risk is captured by a stochastic cost \tilde{c} , to be realized during the implementation of the project. The firm is too small to have direct access to capital markets.⁵ By engaging in relationship lending, the bank obtains information about the distribution of the technological shock and can share this information with otherwise uninformed investors. The bank pools resources from a large number of small investors each endowed with one unit of wealth. Hence, the bank in our model plays a twofold intermediation role: it acquires information about investment opportunities for the use of investors, and channels resources from investors towards firms. Investors can either invest their wealth in a risk-free asset which yields return r , or place their funds with the bank that will invest them in the firm, for a risky return R .

Given the linear technology in capital, the probability that the firm succeeds is increasing in the availability of capital; in turn, the amount of available capital depends on the number of investors who place their funds with the bank. This brings about a typical coordination problem, which under perfect information, presents multiple equilibria (Diamond and Dybvig, 1983). If the technological cost is not too high, there is a high-risk, Pareto dominant equilibrium, where all investors invest; any return set by the bank (with $R \geq r$) can support this equilibrium. Whatever the positive cost, there is also a zero-risk Pareto dominated equilibrium where no investor places its funds in the bank and the project fails.

However, there is no need to argue that the assumption of perfect information is too strong, in particular when applied to highly innovative entrepreneurial firms. If we assume instead that investors observe only a noisy signal of the true technological cost, then their beliefs are no longer common knowledge and the problem can be analyzed as a standard "global game" (Carlsson and Van Damme, 1993). Morris and Shin (1998) have proposed an elegant equilibrium concept that applies to n -player coordination games with noisy information.⁶ In this methodology, the "fundamentals of the economy" are captured by a state variable related to the ability of the economy to cope with external shocks. Under ex-post and ex-ante uncertainty about these fundamentals, the

⁵This can be the case for instance if private investors have no possibility to gauge the technological risk on their own.

⁶The methodology has been applied to study various economic problems involving coordination frictions, such as currency crises, bank runs, credit risk and illiquidity debt default (Morris and Shin, 2001; Morris and Shin, 2003; Morris and Shin, 2004; Morris and Shin, 2009; Bebchuk and Goldstein, 2011).

multiple equilibria specific to the perfect information model vanish, and the coordination problem is pinned-down around a unique “threshold or switching” equilibrium. Our solution builds on this established methodology.⁷ We show that investors’ coordination problem presents a switching equilibrium, where the project succeeds if the technological cost is below a critical value, and fails in the opposite case.

The original contribution of this paper is to show that this critical threshold depends in a nonlinear way on the return R chosen by the bank. The impact of the return on capital on the critical cost is driven by two opposite effects. On the one hand, higher returns make the decision to lend to the bank more appealing and support the collective decision to invest; in turn, this increases the probability of success of the firm. On the other hand, higher interest rates raise the capital cost of the firm and has an adverse effect on the probability of success. We show that there exists an “optimal interest rate” which grants the highest chances of success to the entrepreneurial firm. This interest rate is higher than the risk-free interest rate, but this “abnormal margin” has nothing to do with market power. This result is in line with the empirical findings which show that relationship-lending decreases the probability that firms are credit rationed, however not at a necessarily lower cost of borrowing (see, for example, Bartoli et al., 2013).

Our model shows how banks can mitigate an inefficient credit freeze to the real sector through higher interest rates. A work related to ours is Bebchuk and Goldstein (2011). In their model, however, real sector projects are highly interdependent. If all banks provide credit, all firms do well, and banks can recover their investment; yet if several banks refuse to lend, all firms collapse, and it is rational for all banks to refuse loans. Hence the credit freeze can arise from a coordination failure among banks that would not extend credit to operating firms. Appropriate policy responses can get an economy out of an inefficient credit market freeze. Our work can also be related to several theoretical papers which study bank-firm lending relationships. For example, Rajan (1992) and Petersen and Rajan (1995) model the monitoring role of banks which can control the efforts exerted by entrepreneurs and allow only positive NPV projects to be continued. Uniformed investors, on the other hand, cannot prevent entrepreneurs from continuing unprofitable projects. In our model,

⁷While in existing global games models the state variable has a rather “generic” nature, in this paper, the state variable can be related directly to the technological uncertainty specific to new investment projects.

however, the entrepreneur is implicitly assumed to exert its optimal level of effort. Our focus is on showing how banks can use this lending technology to increase the access to capital of small firms.

The paper is organized as follows. The next section lays out the main assumptions. Section 3 presents the equilibrium solution. The question of how banks determine the optimal interest rate is addressed in Section 4. The last section concludes.

2 Main assumptions

The model is cast as a game between a continuum of small investors, a bank operating in a competitive environment and a small entrepreneurial firm. The firm has no endowment and needs to borrow to invest in a new project. Investors are the only agents endowed with wealth but face extreme information asymmetry if they lend directly to the firm. The bank can reduce this asymmetry by engaging in relationship lending with the firm.

A. The firm

The firm is the owner of a new technology or new productive process that requires capital in order to be implemented. We assume that the firm has no funds and needs to borrow to finance the project. The firm's output is a linear function in the capital used in the production process: $Y = (1 + A)K$, with K being the amount of capital and A a positive parameter characteristic of the marginal product of capital ($1 + A$). Implementation of the project is subject to substantial technological uncertainty. To keep the model as simple as possible, we model this uncertainty as a stochastic technological cost \tilde{c} . For instance, for a given project, the number of design hours or research time can exceed by far the normal value (i.e., the long run average). For analytical convenience, we assume that this shock follows a normal distribution $\tilde{c} \rightsquigarrow N(\bar{c}, \sigma^2)$, with a mean value $\bar{c} > 0$ and a precision $\alpha = 1/\sigma^2$.⁸

Given these assumptions the profit function can be written as:

$$\tilde{\pi} = (1 + A)K - (1 + R)K - \tilde{c} \tag{1}$$

where R is return required by the bank for the loan, with $R \leq A$.

⁸Normally the cost cannot be negative. Hence the mean should be large enough and the variance small enough such that $\Pr[c < 0]$ is negligible.

B. The Bank

The bank operates in a competitive environment, making zero profits on its loans.⁹ It channels funds from investors to entrepreneurs. In this intermediation process, the bank serves two important functions.

Firstly, the bank engages in relationship lending with the firm which allows it to collect private information about the worthiness of the project. More precisely, we assume that the bank, by acquiring knowledge about the sector, the firm or the entrepreneur, is able to observe the true distribution of technological shocks; it will then share this information with (ex-ante uninformed) investors. Investors who want to seize this opportunity become bank's clients, i.e., should they decide to participate to the project, they commit themselves to do it through the intermediation of the bank.

Secondly, the bank pools resources from many small investors who decide to participate to the risky collective project and invest them in the firm. Thus the simplified balance sheet of the bank will record investors' deposits as liabilities and a "massive" loan to the firm as the main asset equivalent to these deposits.

Since we assume that the bank operates in a competitive environment, profits should tend to zero; as a consequence, the bank will apply the same interest rate R on both the loan and deposits. This interest rate is decided by the bank at the onset of the game.

Finally, as argued in the Introduction, we assume that the bank's main goal is to maximize chances that the project succeeds.

C. Investors

There is a continuum of $N = 1$ risk-neutral investors, each endowed with one unit of wealth. They have the choice between placing their funds in the banking sector or investing them in a safe asset which yields a return r . The return promised by the bank is risky and depends on the success of the entrepreneurial firm. If the investment in the firm proves to be successful, investors receive the return R , with $R \geq r > 0$. If the firm fails, investors recover a liquidation value, $v < 1$. We denote the proportion of investors who lend to the bank by ℓ , with $\ell \in [0, 1]$.

⁹Despite the competitive nature of the banking sector, the firm can only borrow from one bank, which reduces our model to a "one bank" model. This assumption is strongly supported by the empirical findings that show how small firms generally borrow from a single, most of the time local, bank (see, for example, Agarwal and Hauswald, 2010; Berger and Schaeck, 2011).

At the outset of the game investors are "uninformed", they have no means of inferring the worthiness of the new project from public information. By entering in a contract with the bank, the latter shares with them information about the true distribution of the technological shock. At that moment the distribution of shocks $\tilde{c} \rightsquigarrow N(\bar{c}, \sigma^2)$ becomes common knowledge.

Once that the shock (technological cost) is realized, investors receive a signal about the true value of c . More precisely, an investor $i \in (0, 1)$ will observe

$$x_i = c + \epsilon_i \quad (2)$$

where $\epsilon_i \rightsquigarrow N(0, \tau^2)$. The precision of the signal is denoted by $\beta = 1/\tau^2$.

They then decide whether to invest in the risky firm via the bank.

Investors' net payoff, contingent upon their individual and joint decisions, as well as the realization of the shock c , can be written as:

$$U = \begin{cases} \text{lend to the bank} & \begin{cases} 1 + R & \text{if } \pi \geq 0 \\ v & \text{if } \pi < 0 \end{cases} \\ \text{invest in safe asset} & r \end{cases}$$

D. Timing

The sequence of decisions and the information structure of the game are depicted in Figure 1.

t=0	t=1	t=2
<ul style="list-style-type: none"> • The firm and the bank enter the credit relationship • Bank decides on R 	<ul style="list-style-type: none"> • Technological shock is realized • Investors observe a noisy signal about the technological shock • Investors decide on lending to the bank or investing in the safe asset • The bank transfers capital to the firm 	<ul style="list-style-type: none"> • Investors receive R if project is successful, and v otherwise

Figure 1: Timing

At time $t = 0$, the firm and the bank enter the credit relationship. The bank observes the distribution of technological shocks and shares the information with investors. It sets the interest rate R .

At time $t = 1$, the technological shock is realized. Investors observe a noisy signal. They decide whether to lend their funds to the bank based on the signals they receive and the return R promised by the bank in case of success of the project. The supply of funds for the project is thus equal to the number of investors that choose the participation strategy, ℓ . The bank transfers the funds to the firm.

At time $t = 2$, the firm makes a positive profit or not (is bankrupt). If the profit is positive, the firm pays to investors the contracted return R ; if not, the firm is liquidated and investors get the residual value v .

3 The Equilibrium

Given the firm's production technology in Equation (1), its demand for funds is infinitely elastic. Thus the equilibrium capital is given by the supply of funds: $K = \ell$ and the firm's profit function can be re-written as:

$$\tilde{\pi} = (A - R)\ell - \tilde{c}. \quad (3)$$

Equation 3 points out the importance of coordination among investors. When the cost is zero (or below), the project succeeds even if only one investor has participated. When $c > (A - R)$, the firm fails even if all investors ($\ell = 1$) have participated. When c lies on the interval $(0, A - R)$, the firm succeeds only if a critical mass of investors participate; but the decision to participate depends on every investor's belief about the beliefs of the others.

The signals x_i that investors receive convey information not only about the technological cost c , but also about the signals that other investors receive. At the extreme, when $c = 0$, an investor should lend no matter what the others do. Consider now an investor receiving a signal slightly above zero. This investor figures out that other investors might have received signals equal to or below zero, thus have lending as a dominant strategy. If this investors has a dominant action to lend, all the other investors who received a signal lower than him have lending as a dominant strategy. Applying the same logic several times, we can establish a boundary well above zero below which investors should lend. At the same time, investors have a dominant action not to lend when $c > (A - R)$, because projects fail even if every investor participates. So when an investor receives a signal slightly below $(A - R)$ he is pessimistic about the probability of success of the firm and prefers not to lend. Again, we can apply a

backward reasoning and establish a boundary well below $(A - R)$ above which investors do not lend. A formal proof, presented in Morris and Shin (1998) and Morris and Shin (2004), shows that these two boundaries coincide, such that the coordination problem admits a unique equilibrium characterized by a "switching strategy" (invest / do not invest) around a critical signal. For brevity, we do not repeat here their argument of the proof.

The equilibrium of the game is thus characterized by two thresholds, a "critical signal" x^* driving investors' decision (invest / do not invest), and a "critical cost" c^* for which the firm's project is right on the edge between failing or not. Proposition 1 states our basic equilibrium result.

PROPOSITION 1. The problem presents a single equilibrium provided that the precision of the signal is large enough, more precisely if the sufficient (not necessary) condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A - R}$ holds. The equilibrium critical cost, below which the firm's project succeeds is characterized by the following equation:

$$c^* = (A - R)\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha}\Phi^{-1}\left(\frac{1 + r - v}{1 + R - v}\right)\right]\right) \quad (4)$$

PROOF. Following standard resolution steps (see Morris and Shin, 2004), the two thresholds, x^* and c^* can be determined as the solution to a system of two equations.

First, when Nature draws a cost c , the *proportion of investors who lend* is equal to the frequency of investors who receive a signal below the *critical signal* x^* :

$$\ell = \Pr(x_i < x^* | c) = \Pr(\epsilon_i < x^* - c) = \Phi(\sqrt{\beta}(x^* - c)) \quad (5)$$

where $\Phi()$ is the c.d.f. of the standard normal distribution.

Turning to the *critical cost* c^* , such as for $c > c^*$ the firm fails and for $c \leq c^*$ the firm succeeds. Given the profit function in Equation (3), c^* can be written: $c^* = (A - R)\ell$. Moreover, based on Equation (5) above, the proportion of investors who lend when the cost is exactly c^* is $\Phi(\sqrt{\beta}(x^* - c^*))$. It follows that the critical cost c^* is implicitly defined by:

$$c^* = (A - R)\Phi(\sqrt{\beta}(x^* - c^*)). \quad (6)$$

This gives us the first equation in c^* and x^* .

Second, given our assumptions about the normality of the distributions of costs and signals, when an investor i receives a signal x_i , his posterior distribution of c is also normal with mean $\left(\frac{\alpha\bar{c} + \beta x_i}{\alpha + \beta}\right)$ and precision $(\alpha + \beta)$. So, for an investor with signal x_i , the probability of failure of the firm is:

$$\Pr(c > c^* | x_i) = 1 - \Phi \left(\sqrt{\alpha + \beta} \left[c^* - \frac{\alpha\bar{c} + \beta x_i}{\alpha + \beta} \right] \right)$$

Among the continuum of investors, there exists one who receives exactly the critical signal x^* ; this individual is indifferent between lending or not to the bank. His indifference condition can be written as:

$$(1 + R) [1 - \Pr(c > c^* | x^*)] + v \Pr(c > c^* | x^*) = (1 + r),$$

which is equivalent to:

$$\Phi \left(\sqrt{\alpha + \beta} \left[c^* - \frac{\alpha\bar{c} + \beta x^*}{\alpha + \beta} \right] \right) = \frac{1 + r - v}{1 + R - v}$$

After some calculations it follows that:

$$x^* - c^* = \frac{\alpha}{\beta}(c^* - \bar{c}) - \frac{\sqrt{\alpha + \beta}}{\beta} \Phi^{-1} \left(\frac{1 + r - v}{1 + R - v} \right). \quad (7)$$

which gives us the second equation in x^* and c^* .

Our *equilibrium* critical thresholds x_E^* and c_E^* are thus the solution to the system of equations (6) and (7). By substituting $(x^* - c^*)$ as given by (7) in (6), we get that c_E^* is the solution to the equation:

$$c^* = (A - R) \Phi \left(\frac{\alpha}{\sqrt{\beta}} \left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1} \left(\frac{1 + r - v}{1 + R - v} \right) \right] \right)$$

Graphically, the equilibrium critical cost (the failure threshold) c_E^* is obtained at the intersection between the 45° line and the scaled-up cumulative normal distribution with mean $\left[\bar{c} + \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1} \left(\frac{1 + r - v}{1 + R - v} \right) \right]$ and standard deviation $\frac{\alpha}{\sqrt{\beta}}$. A single solution is guaranteed when the slope of the right hand-side of Equation (4) less than one, that is: $(A - R) \phi(\cdot) \frac{\alpha}{\sqrt{\beta}} < 1$, where $\phi(\cdot)$ is the p.d.f. of the standard normal distribution. Since $\phi(\cdot) < \frac{1}{\sqrt{2\pi}}$, then the unique

solution exists if the sufficient (not necessary) condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$ holds. *QED.*

The equilibrium critical cost c_E^* is represented at point E in Figure 2, draw for parameters that fulfill the single equilibrium condition.

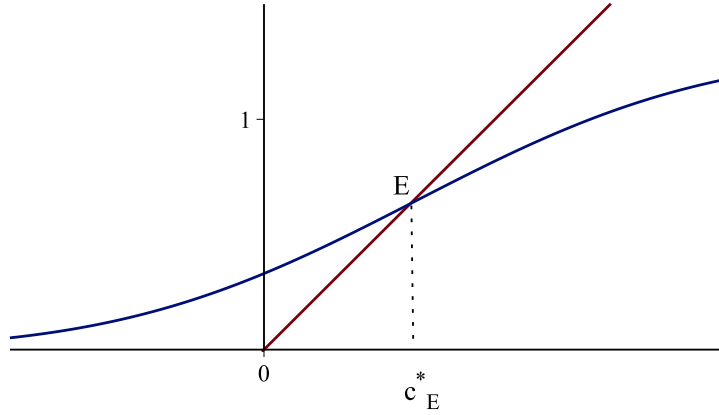


Figure 2: The equilibrium critical cost

Equation 7 shows that the critical signal x_E^* is a linear function in c_E^* . So the uniqueness of c_E^* guarantees uniqueness of the critical signal x_E^* . Furthermore, from Equation (4) we can infer that $0 < c_E^* < (A - R)$. For any cost $c \in (0, c_E^*)$ enough investors participate for the project to succeed; for $c > c_E^*$ not enough investors participate and the project fails; notice that for $c_E^* < c < (A - R)$, the project would succeed if all investors were participating (but they do not). This form of "inefficient credit freeze" is the outcome of investors' failure to coordinate on an otherwise "good" project.

The condition for uniqueness always holds in the special case where *private signals become infinitely precise*, i.e. $\beta \rightarrow \infty$. At this limit, x_E^* and c_E^* converge to the same value, which is given by:

$$c_{\beta \rightarrow \infty}^* = x_{\beta \rightarrow \infty}^* = (A - R)\Phi\left(-\Phi^{-1}\left(\frac{1 + r - v}{1 + R - v}\right)\right) = \frac{(A - R)(R - r)}{1 + R - v} \quad (8)$$

Intuitively, because investors' signals have infinitesimally small noise, in equilibrium all investors should lend when the cost is below c^* and none of them should lend when the cost is above the critical value.

4 The optimal return

4.1 The general case

Our key interest rests in understanding how the return on capital impacts the equilibrium critical cost and, implicitly, the probability of success of the firm. We have argued that the bank sets the return R with the aim of maximizing the probability that the project succeeds. It follows that the bank's optimization problem can be stated as:

$$\max_R \Pr[c < c_E^*]$$

From now on we refer only to the equilibrium critical cost and signal. To avoid excessively complex notation, we can drop the subscript E . The variation in c^* with respect to R is driven by two opposite effects. On the one hand, a higher R should increase the participation rate ℓ in the risky project and the chances that the project succeeds. On the other hand, a higher R raises the capital cost, so it increases the probability that the firm will default on its liabilities. An important result is stated in Proposition 2.

PROPOSITION 2. There exists an optimal return \hat{R} that maximizes the critical cost c^* and this optimal return is higher than the risk-free interest rate r .

PROOF. Starting from Equation (4), and applying the implicit function theorem (see Appendix) we get:

$$\frac{dc^*}{dR} = \frac{\sqrt{\frac{\alpha + \beta}{\beta}} \frac{(1 + r - v)(A - R)}{(1 + R - v)^2} \frac{\phi(\cdot)}{\phi\left(\Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right)\right)} - \Phi(\cdot)}{1 - (A - R) \frac{\alpha}{\sqrt{\beta}} \phi(\cdot)}, \quad (9)$$

where $\Phi(\cdot) = \Phi\left(\frac{\alpha}{\sqrt{\beta}} \left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right) \right]\right)$ and $\phi(\cdot) = \phi\left(\frac{\alpha}{\sqrt{\beta}} \left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1}\left(\frac{1+r-v}{1+R-v}\right) \right]\right)$.

From Equation (4) we know that on the interval $R \in [r, A]$, the function $c^*(R)$ is continuous, positive and lower than $(A - R)$. Moreover, since $R \in [r, A]$, at the extremes we have that $\lim_{R \rightarrow r} c^* = 0$ and $\lim_{R \rightarrow A} c^* = 0$, because $\Phi^{-1}(1) = \infty$ and $\Phi(-\infty) = 0$.

In Equation (9), we can check that $\left[\frac{dc^*}{dR}\right]_{R=A} < 0$. Indeed, the denominator of expression (9) is always positive because $1 - (A - R)\frac{\alpha}{\sqrt{\beta}}\phi(\cdot) \geq 1 - (A - R)\frac{\alpha}{\sqrt{\beta}}\frac{1}{\sqrt{2\pi}} > 0$, given the uniqueness condition $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A-R}$. The sign of derivative is thus the sign of $-\Phi(\cdot)$; for $R = A$, we have $\Phi(\cdot) > 0$. This suffices to prove that $c^*(R)$ admits at least one maximum for $\hat{R} \in (r, A)$. Moreover, because $\lim_{R \rightarrow r} c^* = 0$, this maximum is necessarily higher than the risk-free rate r . *QED*

This result is worth being emphasized. If the coordination risk could be ruled out, for instance if the bank raises funds from a single large investor (instead of many small investors), chances that the firm succeeds (makes a positive profit) are given by the probability $\Pr[c < (A - R)]$ (see Eq.3). Obviously, the competitive bank would maximize these chances if it sets $R = r$. In the presence of the coordination risk, the optimal interest rate is higher than the risk-free rate, but this high interest rate is not the consequence of banks abusing of some form of market power; to the contrary, *the high rate is beneficial to the firm*, insofar as it helps relaxing the funding constraint.

In the Appendix we perform other comparative statics with respect to the equilibrium critical cost. First, we show that a lower risk-free rate, r , prompts more investors to participate to the collective project, and thus raises the probability of success of the firm. This would correspond to a situation when key interest rates of monetary policy are cut; in our model, this would induce more bank lending to the real sector. A similar effect is brought about by a smaller average technological cost faced by the firm, \bar{c} . This would happen in a period of extreme innovation (such as the IT revolution in the late nineties). Finally, a closer lending relationship which translates into the ability of the bank to secure a higher liquidation value, v , also increases the firm's access to finance.¹⁰ Notice that a very favorable macroeconomic environment might have an ambiguous effect on the loan rate R ; indeed, in such a context, the average cost \bar{c} might decline, while at the same time the central bank would push up the short-term interest rate r .

¹⁰Government guarantee programs such as the US Small Business Administration 7(a) Loan Program which guarantees bank loans to small businesses, should have a similar impact on the liquidation value of the project, and thus result in an increased access to finance.

4.2 A numerical example

To bring some additional intuition on our main result, Figure 3 uses a numerical simulation to represent the critical cost c^* as a function of R . Parameter values are $c \sim N(0.5, \frac{1}{10})$, $\epsilon \sim N(0, \frac{1}{1000})$, $r = 0$, $A = 2$ and $v = 0.1$. Under these values, the bank maximization problem admits an optimal \hat{R} equal to 0.71. For this return, the equilibrium critical cost below which the firm succeeds is $c^*(0.71) = 0.581$. This gives us a (maximum) probability of success of the firm of 79%.

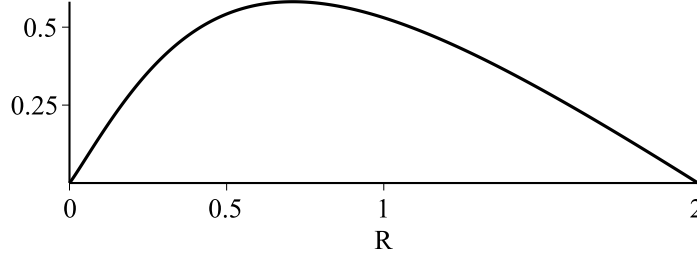


Figure 3: Evolution of threshold equilibrium c_E^* as a function of R

For these optimal \hat{R} and c^* , the equilibrium critical signal below which investors choose to lend to the bank, $x^*(c^*)$, is equal to 0.577 (based on Equation 7). Given this critical signal the actual proportion of investors who decide to participate to the project depends on the realized shock c and is given by $\ell = \Pr[x_i < x^* | c]$. Figure 4 illustrates the proportion of investors who lend as a function of the actual realization of the cost c .

4.3 The special case of infinitely precise private signals

A neat analytical solution for the optimal return set by the bank can be obtained in the special case where private signals are infinitely precise, i.e. $\beta \rightarrow \infty$. In this case, Equation (8) allows us to represent the critical cost $c_{\beta \rightarrow \infty}^*$ as a concave function in R . This critical cost admits a maximum for the optimal \hat{R} implicitly defined by:

$$1 + \hat{R} - v = \sqrt{(1 + r - v)(1 + A - v)} \quad (10)$$

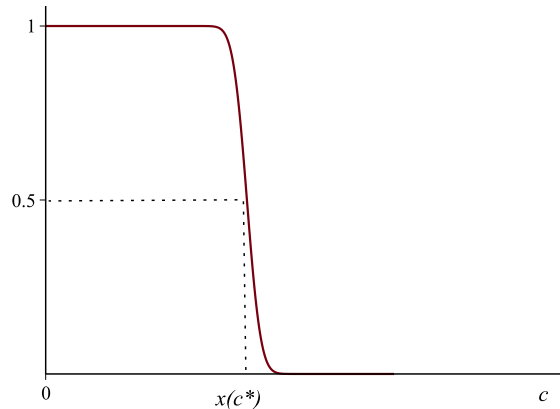


Figure 4: Proportion $l(c)$ of investors

We reach the same conclusion as in the general case: the optimal interest rate is higher than the risk-free rate: $r < \hat{R} < A$.

Furthermore, in the plausible case where $(\hat{R} - v)$, $(r - v)$ and $(A - v)$ are relatively small, taking a log approximation yields:

$$\hat{R} \approx 0.5(r + A).$$

The loan rate R and the risk-free rate r appears to be strongly correlated, with $\partial \hat{R} / \partial r = 0.5$; if the risk free rate increases by one percentage point, banks should rise the optimal loan rates by half percentage point. Interestingly, the residual value v has only a second order effect that can be neglected.

5 Conclusion

In a world where small enterprises are an important engine of growth, understanding how banks can facilitate their access to external funds and contribute to their development is an important issue. This paper has developed a simple model of bank lending which highlights how funding constraints faced by banks affect the cost and availability of external finance for small entrepreneurial firms. In our model the bank performs two important missions: to collect and share specific information about the worthiness of a new project, and to facilitate investors' coordination around investing in the project. We have shown that the possibility that investors fail to coordinate brings about a strategic risk that

compounds its effect with the intrinsic technological risk of the project.

The equilibrium behavior of investors has been analyzed in a standard global games approach. We have determined the equilibrium critical cost, separating the failure / success states of the project. This cost depends in a non-linear way on the return to capital chosen by the bank. Return to capital has two opposite effects on chances of success of the firm. On the one hand, a higher return mitigates the coordination problem by setting more incentive on investors to participate to the collective project; in turn, this increases the volume of available capital and thus fosters chances that the project succeeds. On the other hand, a higher return to investors raises the overall cost of capital, which increases the probability that the firm will default. We show that even in a perfectly competitive environment, banks have no reason to bid interest rates down to the risk-free rate. Actually, there exists an optimal interest rate which maximizes the probability of success of the firm and this interest rate is higher than the risk-free interest rate.

In the special case where the precision of the signal tends to infinity, we have inferred a simple linear relationship between the optimal return and the risk-free rate: whenever the risk free rate increases by one percent, the optimal return must increase, but by less than one percent (by half percentage point in this special case).

It can be seen better now why we choose to study a competitive banking sector. At difference with other papers where the wedge between the risk-free interest rate and the project loan rate is signaling an abuse of market power (Petersen and Rajan, 1995), in our analysis a high loan rate could be imposed by the bank *in the interest of the firm*, as a mean to relax the capital quantity constraint. There is a clear trade-off between the price of capital and the availability of funds. The original measure of availability developed in this paper reflects the complex decision of investors who take into account both the technological risk and the financial-strategic uncertainty.

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A Appendix: Additional comparative statics on the critical cost c^*

In this Appendix we analyze how the equilibrium critical cost c^* varies when R , r , \bar{c} and v change. An increase in c^* is tantamount to an increase in chances that the project succeeds. Start by defining the function:

$$I(c^*, R, r, \bar{c}, v) = c^* - (A - R) \Phi \left(\frac{\alpha}{\sqrt{\beta}} \left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1} \left(\frac{1 + r - v}{1 + R - v} \right) \right] \right) = 0$$

We have:

$$\frac{\partial I}{\partial c^*} = 1 - (A - R) \frac{\alpha}{\sqrt{\beta}} \phi(\cdot) \geq 1 - (A - R) \frac{\alpha}{\sqrt{\beta}} \frac{1}{\sqrt{2\pi}} > 0,$$

given the imposed condition for equilibrium uniqueness $\frac{\alpha}{\sqrt{\beta}} \leq \frac{\sqrt{2\pi}}{A - R}$, and

$$\begin{aligned} \frac{\partial I}{\partial R} &= \Phi(\cdot) - (A - R) \phi(\cdot) \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))} \frac{1 + r - v}{(1 + R - v)^2} \\ \frac{\partial I}{\partial r} &= (A - R) \phi(\cdot) \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))} > 0 \\ \frac{\partial I}{\partial \bar{c}} &= (A - R) \phi(\cdot) \frac{\alpha}{\sqrt{\beta}} > 0 \\ \frac{\partial I}{\partial v} &= (A - R) \phi(\cdot) \frac{\sqrt{\alpha + \beta}}{\sqrt{\beta}} \frac{1}{\phi(\Phi^{-1}(\frac{1+r-v}{1+R-v}))} \frac{r - R}{(1 + R - v)^2} < 0, \end{aligned}$$

where $\Phi(\cdot) = \Phi \left(\frac{\alpha}{\sqrt{\beta}} \left[c^* - \bar{c} - \frac{\sqrt{\alpha + \beta}}{\alpha} \Phi^{-1} \left(\frac{1+r-v}{1+R-v} \right) \right] \right)$. By the implicit function theorem we can write:

$$\begin{aligned} \frac{dc^*}{dR} &= - \frac{\partial I / \partial R}{\partial I / \partial c^*} \\ \frac{dc^*}{dr} &= - \frac{\partial I / \partial r}{\partial I / \partial c^*} < 0 \\ \frac{dc^*}{d\bar{c}} &= - \frac{\partial I / \partial \bar{c}}{\partial I / \partial c^*} < 0 \\ \frac{dc^*}{dv} &= - \frac{\partial I / \partial v}{\partial I / \partial c^*} \frac{\partial c^*}{\partial v} > 0. \end{aligned}$$